

MATHEMATICS

Time allowed: 1 hour 30 minutes

- All answers should be written in the answer books provided, including any diagrams, graphs or sketches. Graph paper is not required.
 - Answer **all** questions in Section A and **two** questions from Section B.
 - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
 - Statistical tables will be provided. Note that the tables refer to the **right-hand** tails of the distributions, that is, probabilities of the form $p = \mathbb{P}(X \geq x)$ where X is a random variable and x an **upper** percentage point of its distribution.
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Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a) $\frac{(2x + \sqrt{3})^2 + (x\sqrt{3} - 2)^2}{x^2 + 1}$ [3 marks]

(b) $\frac{15}{\sqrt{12} - \sqrt{7}} - 6\sqrt{3} - 3\sqrt{7}$ [3 marks]

(c) $2 \log_6 10 + \log_6 3 - 3 \log_6 5 + \log_6 15$ [6 marks]

2. Under suitable conditions, bacterial growth may be modelled as follows: at time t hours, the number of bacteria in a sample is $N = Ce^{at}$ where C and a are constants.

(a) Explain the meaning of the constant C in this context. [1 mark]

(b) Suppose that the number of bacteria in the sample doubles every 5 hours. Find the value of a , to four decimal places. [4 marks]

(c) Find the time at which the sample contains 10 times the initial number of bacteria. [3 marks]

3. Let $f(x) = 2x^3 + 3x^2 - 36x$, for all real values of x .

(a) Find the range of values of x for which $f(x)$ is a decreasing function. [5 marks]

(b) Find the range of values of x for which $f'(x)$ is a decreasing function. [3 marks]

4. In the following statements A and B, x and y are real numbers.

A If $x = y$, then $(x - 2y)^2 = (4x - 3y)^2$.

B If $(2y - x)^2 = (4x - 3y)^2$, then $x = y$.

In both cases, identify whether the statement is true or false. Justify your answer by giving a proof (if true), or a counterexample (if false). [5 marks]

5. Find the indefinite integral $\int(x^2 - x^{-2} + 2x^{-\frac{2}{3}} - 2x^{\frac{2}{3}})dx$. [4 marks]

6. Points A and B have position vectors \mathbf{a} and \mathbf{b} . Point C is the midpoint of the line OB (where O is the origin). Point D lies on the line AC such that $AD : DC$ is 2:1.

(a) Find the vector \mathbf{AC} and the position vector of D in terms of \mathbf{a} and \mathbf{b} . [5 marks]

(b) Point P is the midpoint of the line AB . Show that D lies on the line OP , and find the ratio $OD : DP$. [4 marks]

7. Line L_1 has equation $y - 1 = 2x$. Line L_2 is perpendicular to L_1 , intersecting the y -axis at $(0, 6)$, and line L_3 is parallel to L_1 , intersecting the x -axis at $(2, 0)$.

(a) Write down the equations of L_2 and L_3 . [3 marks]

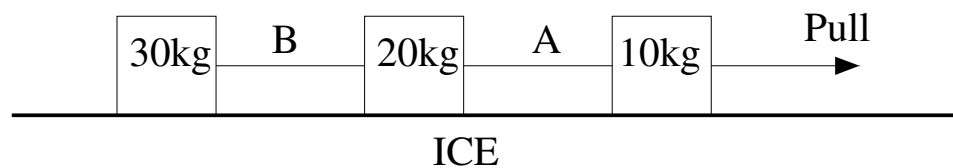
(b) Find the points at which L_2 intersects L_1 and L_3 . [4 marks]

(c) Sketch the lines L_1 , L_2 , and L_3 on the xy -plane. [3 marks]

(d) Line L_4 passes through points $(2, 0)$ and $(0, 1)$. Calculate the area enclosed between the lines L_1 , L_2 , L_3 , and L_4 . [4 marks]

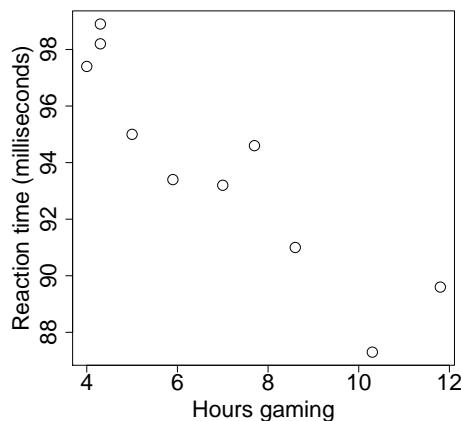
Section B

8. A curve has equation $y = f(x)$, where $f(x)$ is a cubic polynomial. The curve crosses the x -axis at the origin O , and at points A and B equidistant from O . Point A has positive x -coordinate. In addition, $f''(1) = 3$, and the area enclosed between the curve and the line OA is equal to 2.
- (a) Show that $f(x) = \frac{1}{2}x^3 - ax$, where a is a constant whose value is to be found. [10 marks]
 - (b) C is the stationary point of $y = f(x)$ with positive x -coordinate. Find the coordinates of C , and determine the nature of this stationary point. [5 marks]
 - (c) Sketch on the xy -plane the graph of $y = f(x)$, indicating also the points A and C . [3 marks]
 - (d) Find the area between the curve $y = f(x)$ and the lines OC and CA . [2 marks]
9. Three blocks are being pulled horizontally on a frictionless horizontal surface of ice using horizontal ropes aligned with the direction of pull (as shown in the diagram below). Assume that the pull is of magnitude of 125 newtons.



- (a) (i) Considering the three blocks as a single composite object, sketch a diagram showing all the forces acting on it. [2 marks]
- (ii) Hence compute the acceleration of the three blocks across the ice. [2 marks]
- (b) (i) Sketch a diagram showing all the forces acting on each individual block. [6 marks]
- (ii) Hence compute the tensions in ropes A and B . [8 marks]
- (iii) Using the result of part (b) (ii), calculate the acceleration of the 20kg block due to the two tensions, and verify that it is consistent with the result of part (a) (ii). [2 marks]

10. (a) A year before a plant survey, 30% of a population of flowers of a particular species were infected with a known virus. In the current plant survey, a biologist collects 20 flowers from a test site to assess whether the proportion infected has increased since a year ago.
- (i) Write down suitable hypotheses for this test and state a suitable test statistic the biologist could use. [2 marks]
 - (ii) Using a 5% significance level, find the critical region for this test. [3 marks]
 - (iii) What is the probability of a type I error, and what does it mean in this case? [2 marks]
 - (iv) If 11 of the flowers are found to be infected, what is the p-value in this case? What is the conclusion? [3 marks]
- (b) In an experiment to assess the effect of playing a particular computer game on participants' reaction times on a test, the following plot was produced:



- (i) Describe and interpret the correlation between the number of hours of gaming and the reaction time in milliseconds. [3 marks]
- (ii) The regression equation in this case is

$$\text{Reaction time} = 102.7 - 1.28 \times \text{Hours}.$$

Interpret the gradient term and explain why it is reasonable for the regression line to have a non-zero intercept in this case. [3 marks]

- (iii) What would you predict the reaction time to be for somebody who had played for five and a half hours? [2 marks]
- (iv) Why is it not meaningful to use these data to predict somebody's reaction time after 100 hours of game play? [2 marks]