

## MATHEMATICS

**Time allowed: 1 hour 30 minutes**

- All answers (including any diagrams, graphs or sketches) should be written on paper, and scanned into a **single** PDF file. Graph paper is not required.
  - Answer **all** questions in Section A and **two** questions from Section B.
  - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
  - Statistical tables will be provided. Note that the tables refer to the **right-hand** tails of the distributions, that is, probabilities of the form  $p = \mathbb{P}(X \geq x)$  where  $X$  is a random variable and  $x$  an **upper** percentage point of its distribution.
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Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a)  $\frac{2}{\frac{1}{a} - \frac{1}{a+2}} - a^2$  [3 marks]

(b)  $\frac{3}{(3 - \sqrt{12})^2} - 4\sqrt{3}$  [5 marks]

(c)  $\ln(e^{2x} + 2e^x) - \ln(2 + e^x)$  [3 marks]

2. Solve the following equations for  $x$ , giving the answer to 3 decimal places:

(a)  $5^x = 6$ ; [2 marks]

(b)  $(1 - 2^x)^2 = 8$ . [4 marks]

3. Solve the equation

$$6 \cos^2 \theta = 5 - \sin \theta$$

for values of  $\theta$  between  $0^\circ$  and  $360^\circ$ . [8 marks]

4. In the binomial expansion of  $(a - 5x)^{10}$  in powers of  $x$ , the coefficient of  $x^6$  is equal to 5250. Assuming  $a > 0$ , find the value of  $a$ . [4 marks]

5. Find the range of values of  $x$  for which  $x^2 + 3 \leq -14(x + 3)$ . [6 marks]

6. A curve  $C$  has equation  $x^2 + y^2 - 10x + 6y = 15$ . Show that  $C$  is a circle, and find its centre and radius. [6 marks]

7. In the following statements A and B,  $x$  is a real number.

A If  $x^2 - 5x + 6 = 0$ , then  $x = 3$ .

B If  $x = 3$ , then  $x^2 - 5x + 6 = 0$ .

In both cases, identify whether the statement is true or false. Justify your answer by giving a proof (if true), or a counterexample (if false). [5 marks]

8. A curve  $C$  has equation  $y = 2x^{34} + 3x^{23}$ .

(a) Find the equation of the line tangent to  $C$  at  $x = -1$ . [4 marks]

(b) Find the area enclosed between the curve  $C$ , the  $x$ -axis, and the lines  $x = 0$  and  $x = 1$ . [3 marks]

9. Points  $A$  and  $B$  have position vectors  $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$  and  $\mathbf{b} = 5\mathbf{i} + \mathbf{j}$ .

(a) Calculate the distance between  $A$  and  $B$ . [4 marks]

(b) Point  $C$  is the midpoint of the line  $AB$ . Find, in terms of  $\mathbf{i}$  and  $\mathbf{j}$ , the position vector of  $C$ . [3 marks]

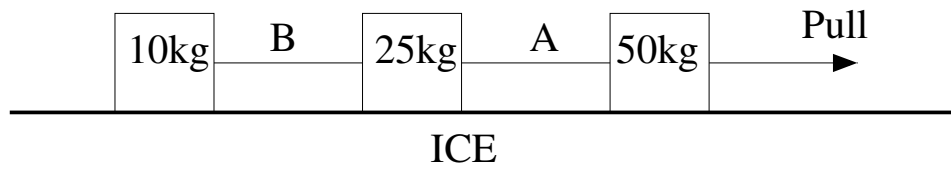
Section B

10. Consider the function

$$f(x) = 4x^3 - 8x - x^4.$$

- (a) Show that  $(1, -5)$  is a stationary point of the curve  $y = f(x)$ . [2 marks]
- (b) Find the other stationary points of  $y = f(x)$ , stating your result using exact values simplified as far as possible. Make sure to indicate your method clearly, showing all working. [9 marks]
- (c) Find the second derivative of  $f(x)$ , and hence determine the nature of each stationary point of the curve  $y = f(x)$ . [7 marks]
- (d) Find the largest value of  $f(x)$ , explaining your reasoning. [2 marks]

11. Three blocks are being pulled horizontally on a frictionless horizontal surface of ice using horizontal ropes (of negligible mass) aligned with the direction of the pull, as shown in the diagram below. Assume that the pull is of magnitude of  $100N$ .



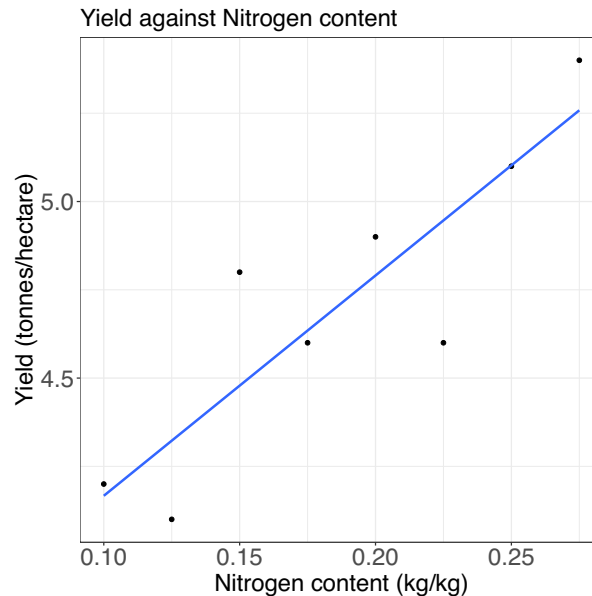
- (a) (i) Considering the three blocks as a composite object, sketch a free body diagram for it, indicating all the forces acting on the object. [2 marks]
- (ii) Hence compute the acceleration of the three blocks across the ice. [2 marks]
- (b) (i) Sketch a separate free body diagram for each individual block (showing all the forces acting on the block). [3 marks]
- (ii) Hence compute the tensions in ropes  $A$  and  $B$ . [8 marks]
- (iii) Using the result of part (b) (ii), find the acceleration of the 25kg block due to the two tensions, and verify that it is consistent with the result of part (a) (ii). [5 marks]

In your answers to (a)(ii) and (b) (ii)-(iii), explain carefully your reasoning, referring to the relevant law of motion.

12. (a) In an experiment to assess the effect of nitrogen content in fertiliser on crop yield, 8 fertilisers were assessed. The data are presented in the table below:

Nitrogen content (kg/kg)	0.1	0.125	0.15	0.175	0.2	0.225	0.25	0.275
Crop yield (tonnes/hectare)	4.2	4.1	4.8	4.6	4.9	4.6	5.1	5.4

Here is a graph of the data including the regression line:



The equation of the regression line is  $Y = 3.54 + 6.24X$ .

- (i) Describe briefly the relevant features of the graph, including the use of the word *correlation*. [3 marks]
  - (ii) Explain clearly what the two numbers in the equation of the regression line mean in the given context. [4 marks]
  - (iii) What is the predicted yield for a fertiliser with a nitrogen content of 0.21 kg/kg? [2 marks]
  - (iv) Suppose that for another crop, the yield is generally higher (over the same range of nitrogen content values), but changes in nitrogen content have less effect on yield. How would you expect the regression line to differ from the one considered above? [2 marks]
- (b) Researchers wish to estimate the proportion of fish with a particular genetic trait in a lake. They sample 20 fish from the lake (with replacement) and find that in the sample, 6 of the fish have the genetic trait.
- (i) If  $X$  is the number of sampled fish having the genetic trait, and  $p$  is the proportion of fish in the lake with the trait, what is the distribution of  $X$ ? [2 marks]
  - (ii) Carry out a hypothesis test to assess whether the proportion of fish in the lake with the trait is less than 50%: formulate the relevant hypotheses in terms of the parameter  $p$  given in (i), calculate the P-value, and state your conclusion using 5% significance level. [7 marks]